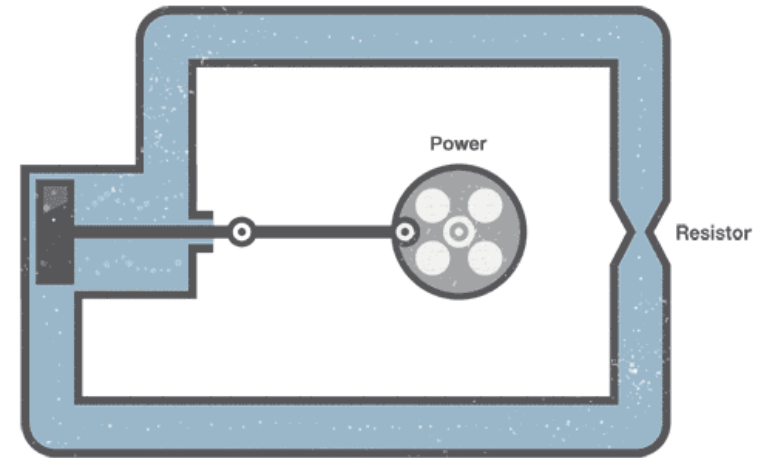




Alternating Current: The Water Analogy



Oscilaciones Forzadas Corriente Alterna

Fuente de CA

R en circuito de CA

C en circuito de CA

L en circuito de CA

Circuito RLC en alterna

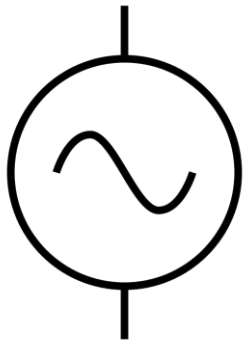
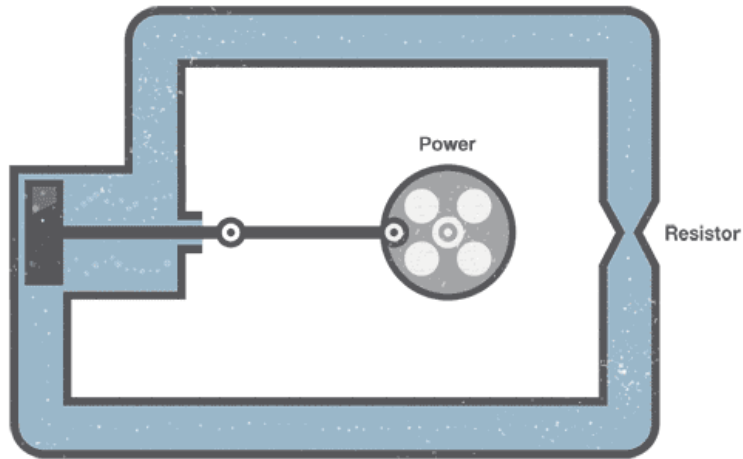
- Impedancia

- Potencia

- Resonancia

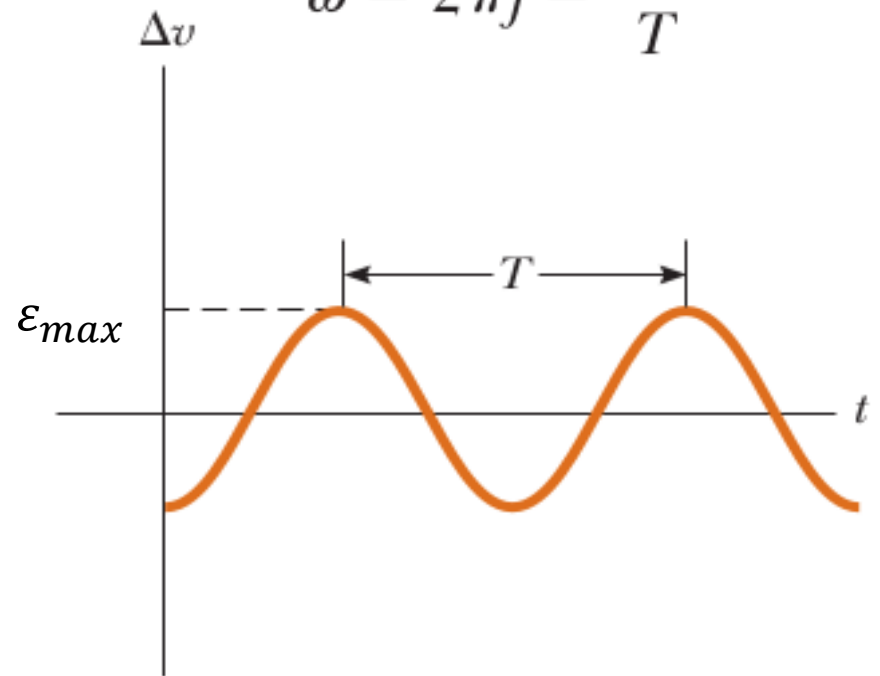
Fuente de CA

Alternating Current: The Water Analogy

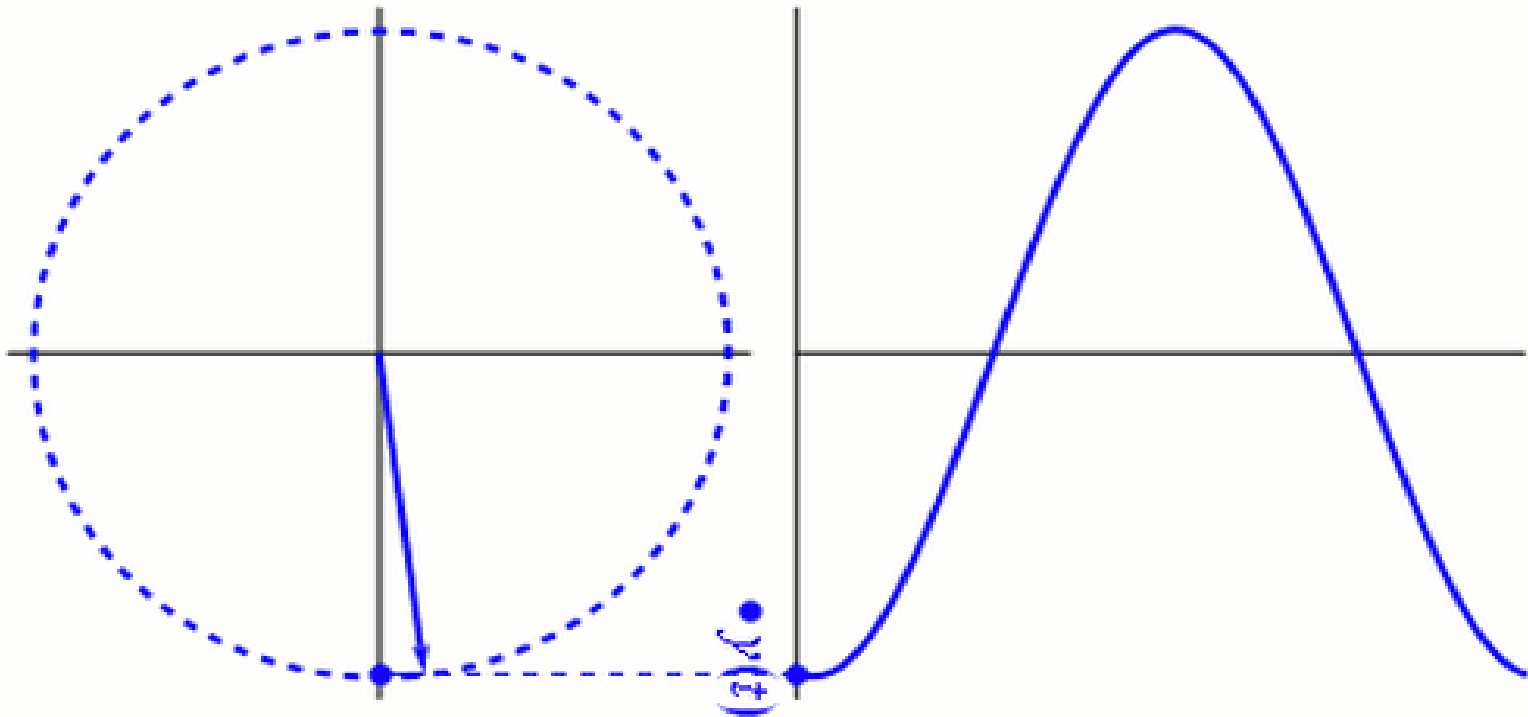


$$\varepsilon(t) = \varepsilon_{max} \text{sen}(\omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

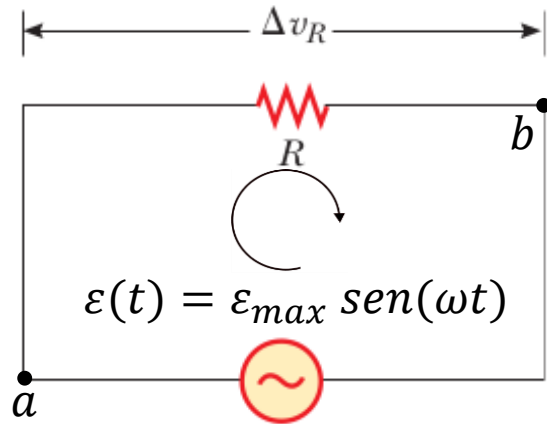


Fasores – Diagrama de Fasores



- Vector que rota en sentido anti-horario.
- Representación abreviada de una función sinusoidal, con una determinada frecuencia
- Su modulo es el valor máximo de la magnitud representada.
- Su componente y da el valor instantáneo de la magnitud.
- 2 magnitudes en fase giran juntas
- Al ángulo de desfase es el ángulo entre los 2 vectores

R en circuito de CA

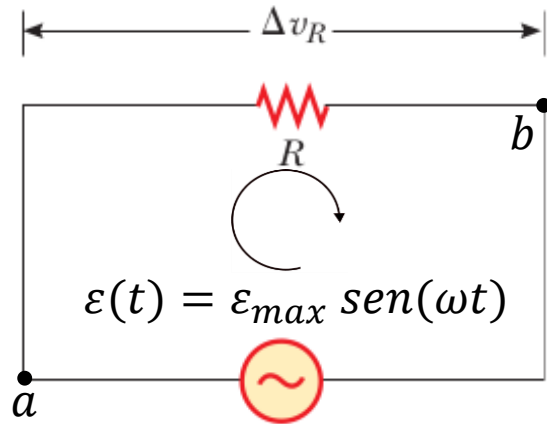


$$(V_a - V_b) + (V_b - V_a) = \varepsilon(t) - iR(t) \quad R = 0$$

$$i_R(t) = \frac{\varepsilon(t)}{R} = \frac{\varepsilon_{max}}{R} \text{sen}(\omega t) = I_{max} \text{sen}(\omega t)$$

$$\Delta v_R(t) = iR(t) \quad R = I_{max} R \text{sen}(\omega t) = \Delta, max \text{sen}(\omega t)$$

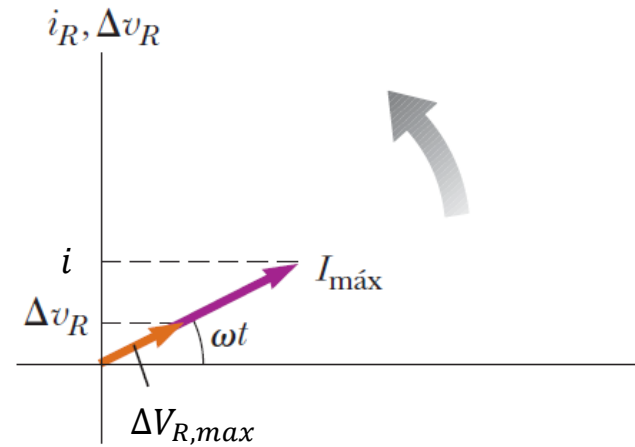
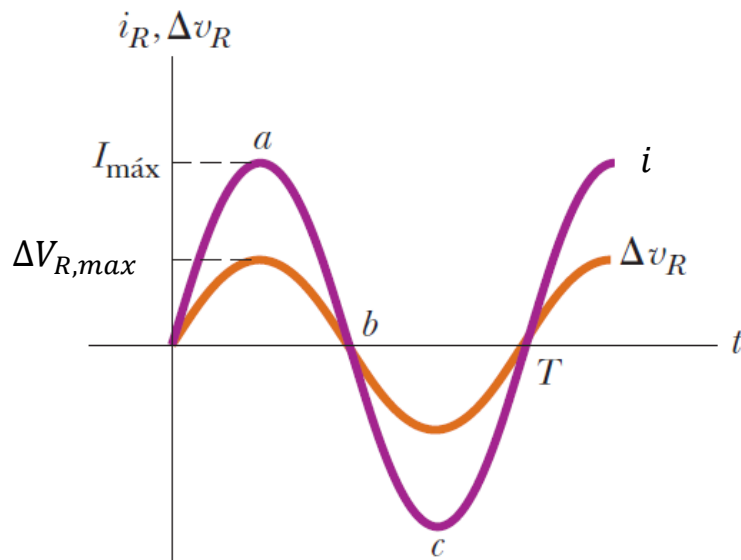
R en circuito de CA



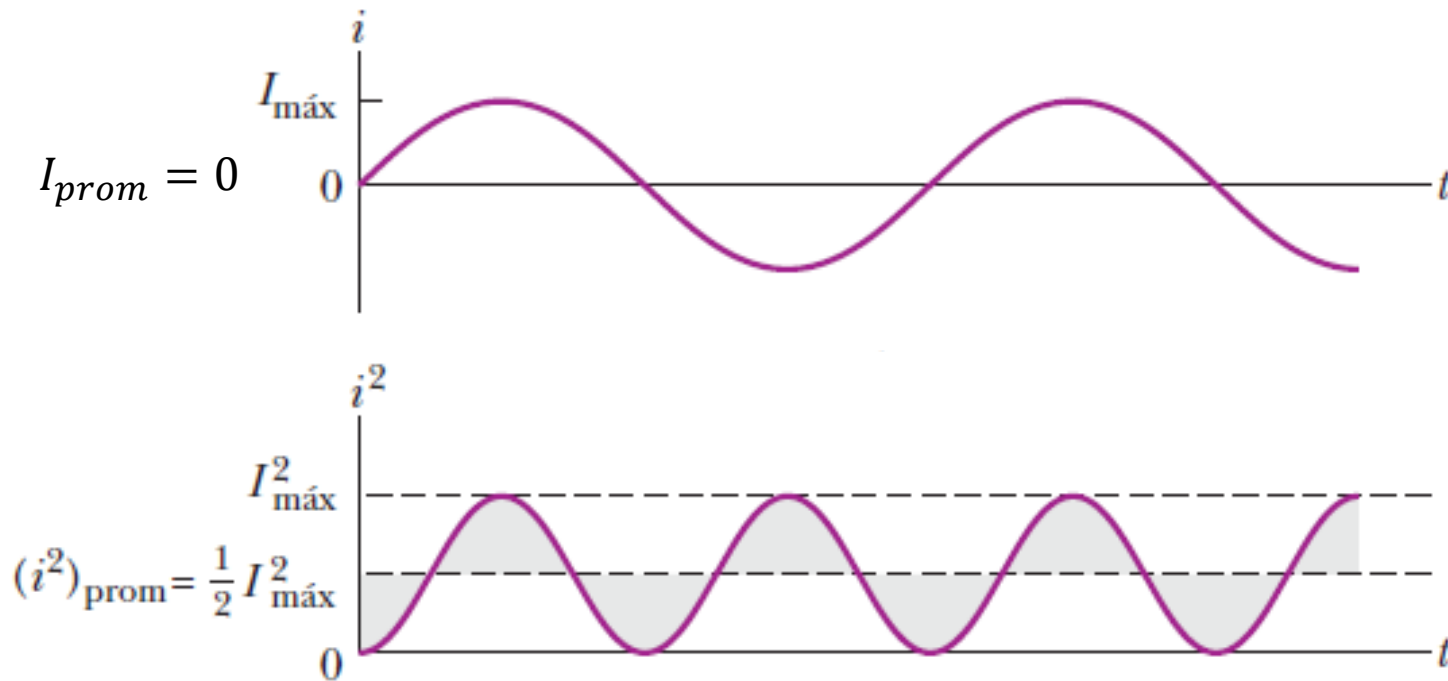
$$\Delta v_R(t) = \Delta V_{R,max} \text{sen}(\omega t)$$

$$i_R(t) = I_{max} \text{sen}(\omega t)$$

$$\Delta V_{R,max} = I_{max} R$$



Valores eficaces

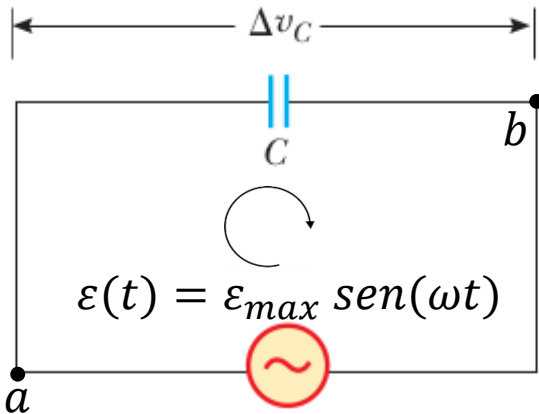


$$I_{ef} = \sqrt{(i^2)_{prom}} = \sqrt{\frac{I_{max}^2}{2}}$$

$$I_{ef} = \frac{I_{max}}{\sqrt{2}}$$

$$\Delta V_{ef} = \frac{\Delta V_{max}}{\sqrt{2}}$$

C en circuito de CA



$$(V_a - V_b) + (V_b - V_a) = \varepsilon(t) - \Delta v_C(t) = 0$$

$$\Delta v_C(t) = \varepsilon_{max} \text{sen}(\omega t) = \Delta V_{C,max} \text{sen}(\omega t) = \frac{q(t)}{C}$$

$$\text{como: } i_C(t) = \frac{dq}{dt}$$

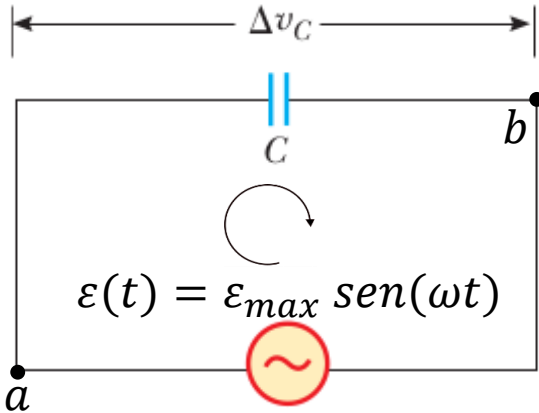
$$i_C(t) = \frac{d}{dt} (C \Delta V_{C,max} \text{sen}(\omega t)) \quad i_C(t) = \boxed{C \omega \Delta V_{C,max}} \cos(\omega t)$$

↓
 I_{max}

$$i_C(t) = I_{max} \cos(\omega t) = I_{max} \text{sen} \left(\omega t + \frac{\pi}{2} \right)$$

$$\Delta V_{C,max} = \frac{1}{\omega C} I_{max} = X_C I_{max} \quad X_C = \text{Reactancia capacitiva}$$

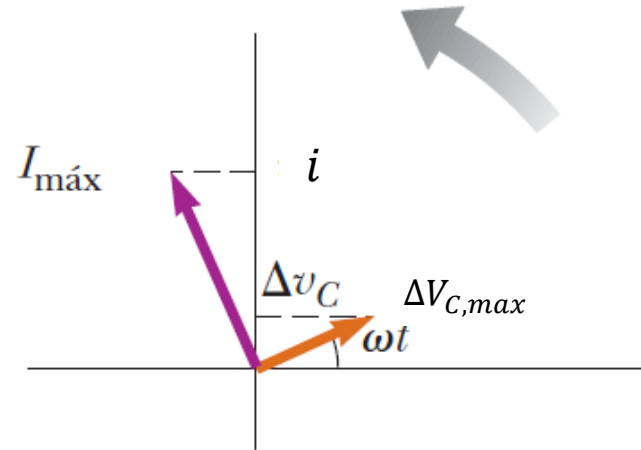
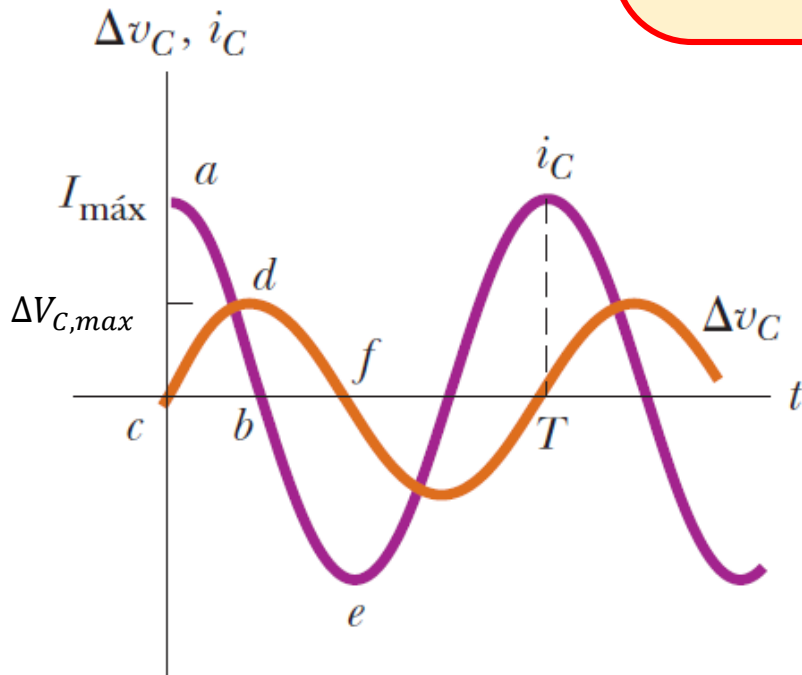
C en circuito de CA



$$\Delta v_C(t) = \Delta V_{C,max} \text{sen}(\omega t)$$

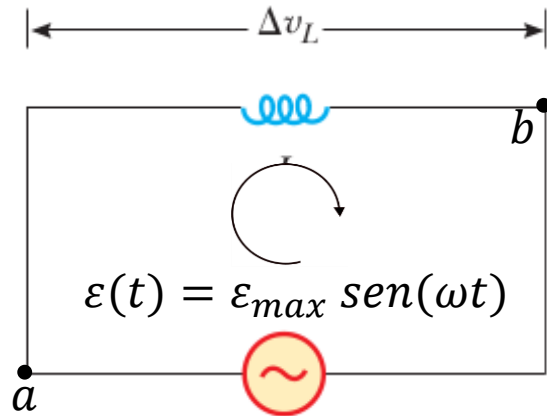
$$i_C(t) = I_{max} \text{sen}\left(\omega t + \frac{\pi}{2}\right)$$

$$\Delta V_{C,max} = I_{max} X_C \quad \text{con} \quad X_C = \frac{1}{\omega C}$$



En C, la tensión se atrasa $\frac{\pi}{2}$ respecto a la corriente

L en circuito de CA



$$(V_a - V_b) + (V_b - V_a) = \varepsilon(t) - \Delta v_L(t) = 0$$

$$\Delta v_L(t) = \varepsilon_{max} \text{sen}(\omega t) = \Delta V_{L,max} \text{sen}(\omega t)$$

$$\Delta v_L(t) = L \frac{di_L}{dt} \Rightarrow \int di_L = \frac{1}{L} \Delta V_{L,max} \int \text{sen}(\omega t) dt$$

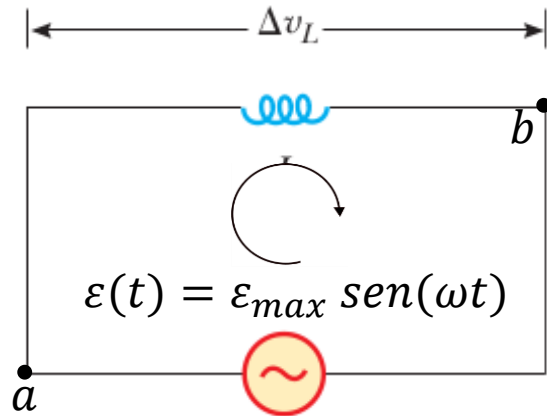
$$i_L(t) = -\frac{\Delta V_{L,max}}{\omega L} \cos(\omega t)$$

$\rightarrow I_{max}$

$$i_L(t) = -I_{max} \cos(\omega t) = I_{max} \text{sen}\left(\omega t - \frac{\pi}{2}\right)$$

$$\Delta V_{L,max} = \omega L I_{max} = X_L I_{max} \quad X_L = \text{Reactancia inductiva}$$

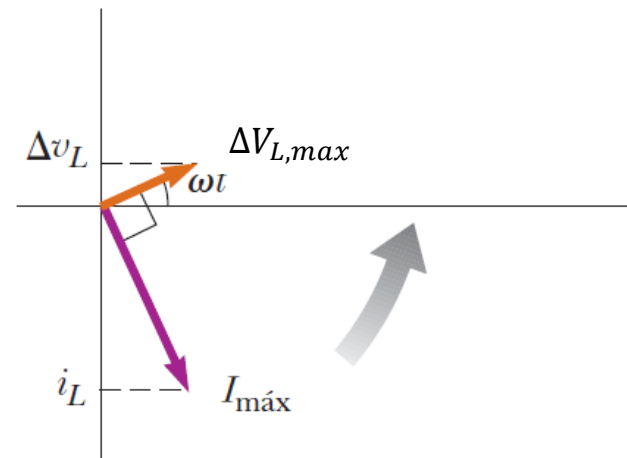
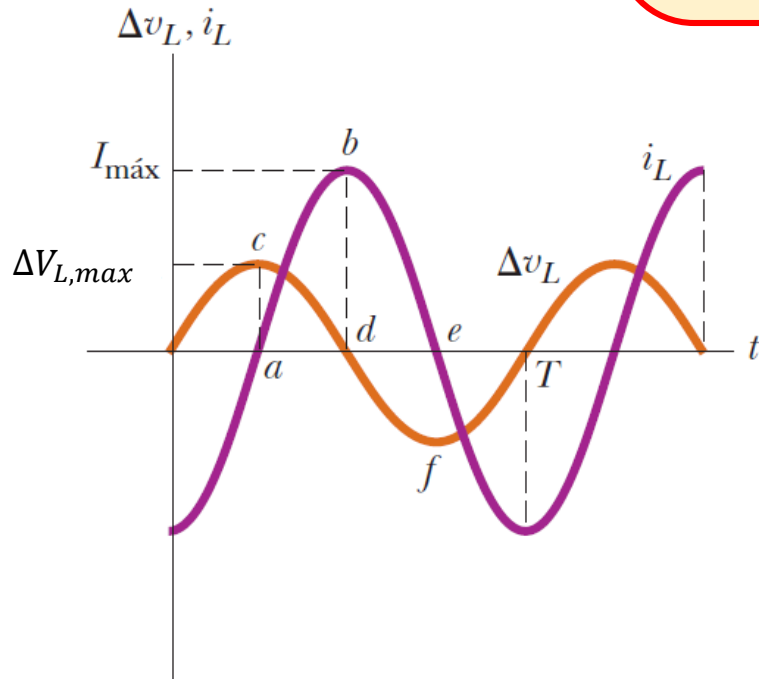
L en circuito de CA



$$\Delta v_L(t) = \Delta V_{L,max} \text{sen}(\omega t)$$

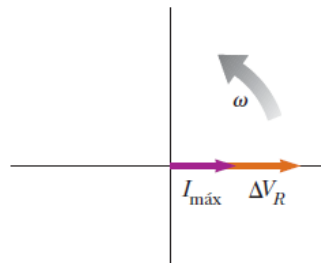
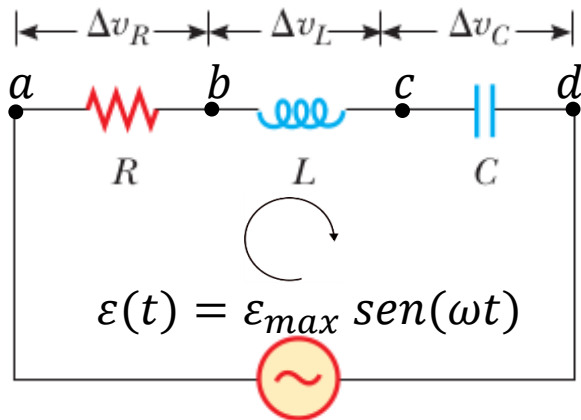
$$i_L(t) = I_{max} \text{sen}\left(\omega t - \frac{\pi}{2}\right)$$

$$\Delta V_{L,max} = I_{max} X_L \quad \text{con} \quad X_L = \omega L$$

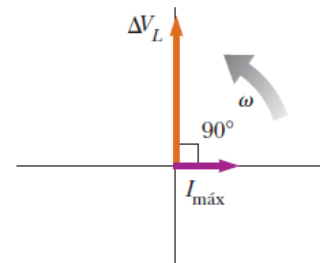


En L, la tensión se adelanta $\frac{\pi}{2}$ respecto a la corriente.

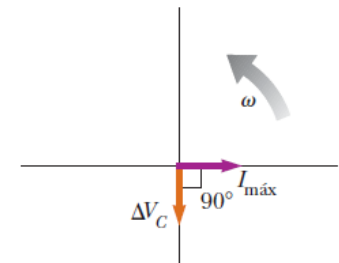
Circuito RLC en CA



a) Resistor

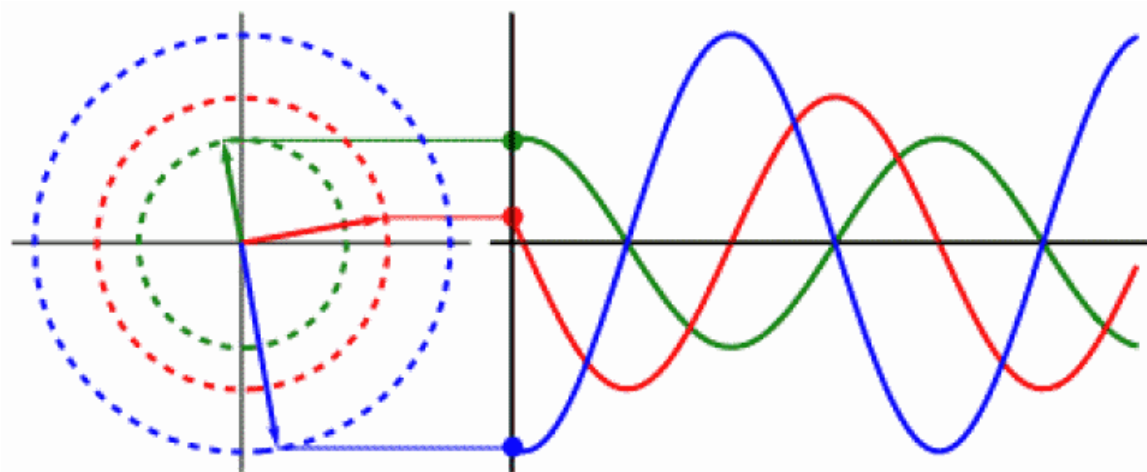


b) Inductor



c) Condensador

$$\begin{aligned}
 (V_a - V_d) + (V_b - V_a) + (V_c - V_b) + (V_d - V_c) &= 0 \\
 \varepsilon(t) - \Delta v_R(t) - \Delta v_L(t) - \Delta v_C(t) &= 0 \\
 \varepsilon(t) &= \Delta v_R(t) + \Delta v_L(t) + \Delta v_C(t)
 \end{aligned}$$

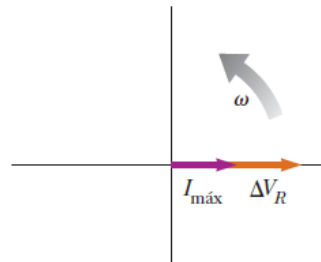
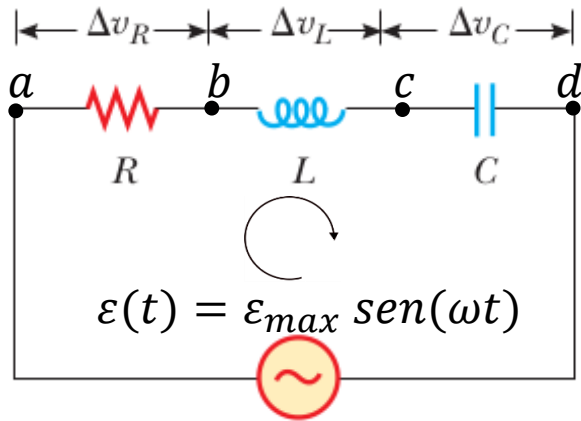


$$\Delta v_L$$

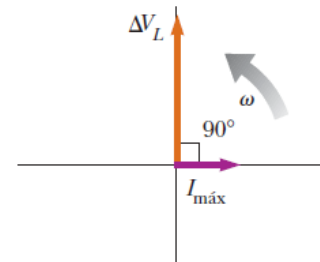
$$\Delta v_R$$

$$\Delta v_C$$

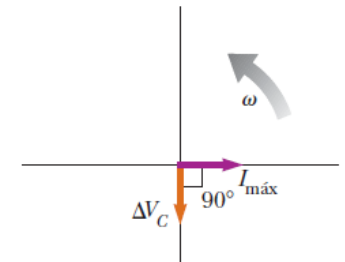
Circuito RLC en CA



a) Resistor



b) Inductor



c) Condensador

$$\varepsilon(t) = \varepsilon_{max} \text{sen}(\omega t)$$

$$\Delta v_R(t) = \Delta v_{R,max} \text{sen}(\omega t) \quad \rightarrow \quad \Delta v_{R,max} = I_{max} R$$

$$\Delta v_L(t) = \Delta V_{L,max} \text{sen}\left(\omega t + \frac{\pi}{2}\right) \quad \rightarrow \quad \Delta v_{C,max} = I_{max} X_L$$

$$\Delta v_C(t) = \Delta V_{C,max} \text{sen}\left(\omega t - \frac{\pi}{2}\right) \quad \rightarrow \quad \Delta v_{L,max} = I_{max} X_C$$

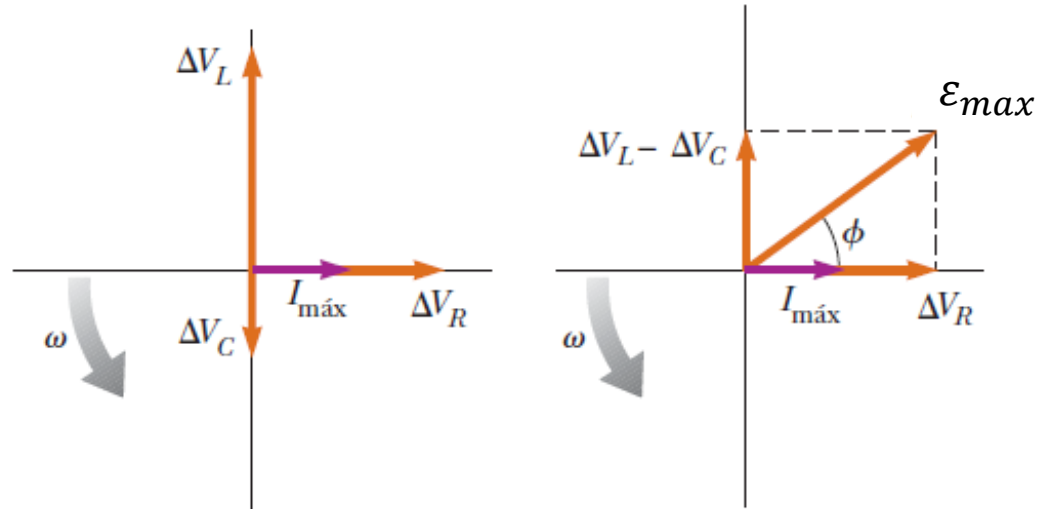
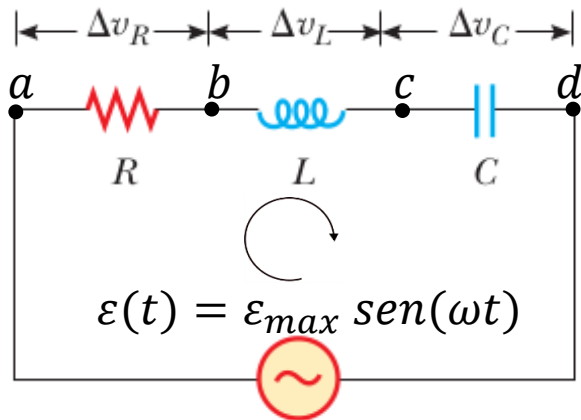
$$i(t) = I_{max} \text{sen}(\omega t + \phi)$$

$$I_{max} = ?$$

$$\phi = ?$$

La corriente $i(t)$ va a ser la misma en todos los elementos, ya que están en serie.

Circuito RLC en CA - Impedancia



$$\varepsilon_{max} = \sqrt{(\Delta V_{R,max})^2 + (\Delta V_{L,max} - \Delta V_{C,max})^2} = \sqrt{(I_{max}R)^2 + (I_{max}X_L - I_{max}X_C)^2}$$

$$\varepsilon_{max} = I_{max} \sqrt{R^2 + (X_L - X_C)^2}$$

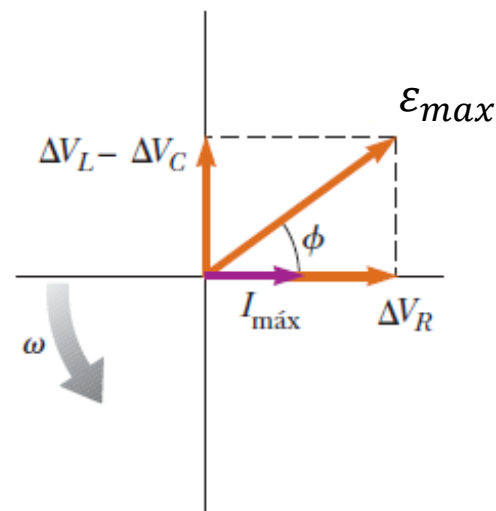
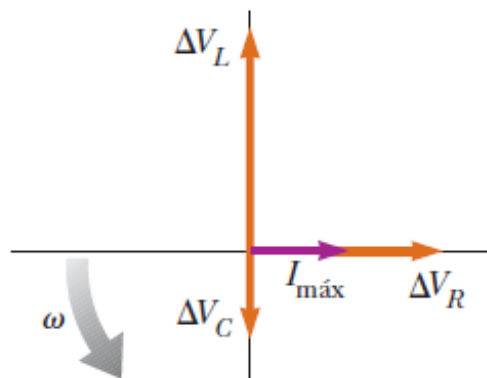
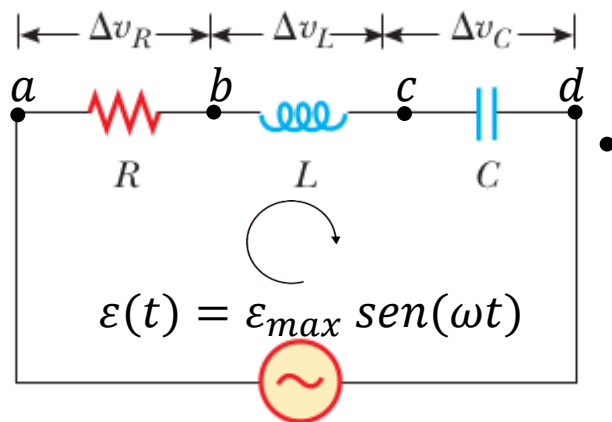
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \text{Impedancia}$$

$$I_{max} = \frac{\varepsilon_{max}}{Z}$$

o

$$I_{ef} = \frac{\varepsilon_{ef}}{Z}$$

Circuito RLC en CA – Angulo ϕ



$$\tan(\phi) = \frac{\Delta V_{L,max} - \Delta V_{C,max}}{\Delta V_{R,max}} = \frac{I_{max} X_L - I_{max} X_C}{I_{max} R} = \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

De esta forma ϕ esta definido desde la corriente hacia la tension

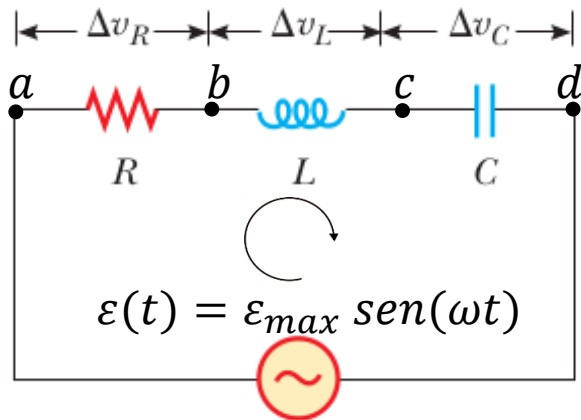
$\phi > 0 \Rightarrow X_L > X_C \rightarrow$ Circuito Inductivo

$\phi < 0 \Rightarrow X_L < X_C \rightarrow$ Circuito Capacitivo

$\phi = 0 \Rightarrow X_L = X_C \rightarrow$ Circuito totalmente

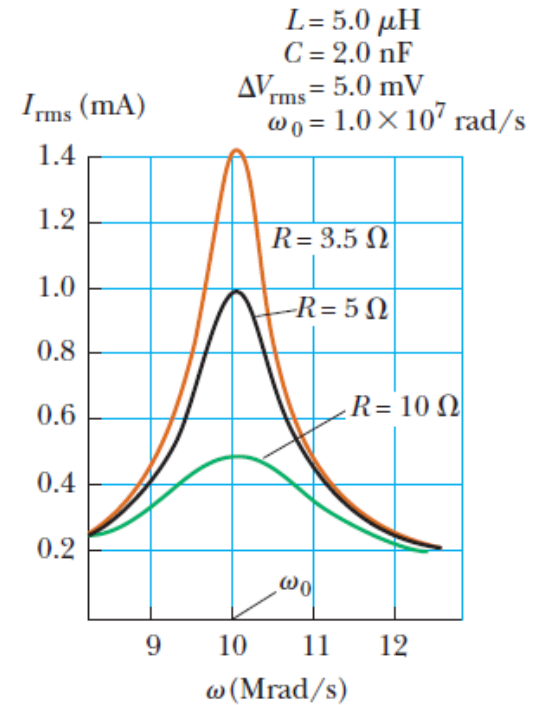
Resistivo o en Resonancia

Circuito RLC en CA en resonancia



Circuito en resonancia:

$$\begin{aligned}
 X_L &= X_C \\
 \Delta V_{L,max} &= \Delta V_{C,max} \\
 \phi &= 0 \\
 Z &= R
 \end{aligned}$$



Resonancia:

$$\omega L = \frac{1}{\omega C}$$

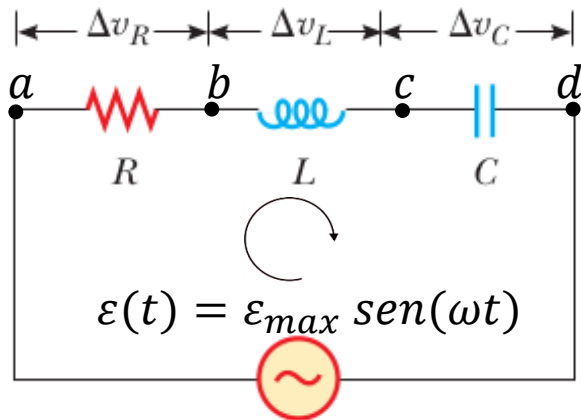
Cambiando $L \rightarrow L = \frac{1}{\omega^2 C}$

Cambiando $C \rightarrow C = \frac{1}{\omega^2 L}$

Cambiando $\omega \rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

} Frecuencia de resonancia

Circuito RLC - Potencia



L y C no disipan energí, solo la almacenan y la devuelven.

Potencias instantáneas:

$$P_{\varepsilon}(t) = \varepsilon(t)i(t)$$

$$P_R(t) = \Delta v_R(t)i(t)$$

No es practico su uso

Lo conveniente es el uso de los valores promedio de potencia en termino de los valores eficaces de corriente y tensión

$$\overline{P_R} = R \frac{I_{max}^2}{2}$$

$$\overline{P_{\varepsilon}} = \frac{\varepsilon_{max} I_{max}}{2} \cos(\phi)$$

$$\overline{P_R} = R I_{ef}^2$$

$$\overline{P_L} = 0$$

$$\overline{P_{\varepsilon}} = \varepsilon_{ef} I_{ef} \cos(\phi)$$

$$\overline{P_C} = 0$$

$$\cos(\phi) = \frac{R}{Z}$$